RAN-2003000204020031

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S.Y.B.Sc. (Sem. - IV) Examination April - 2023

MTH- 401- Mathematics (Paper-VIII)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) **નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.**
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S.Y.B.Sc. (Sem. - IV)**

Name of the Subject :

☛ **MTH- 401- Mathematics (Paper-VIII)**

Subject Code No.: **2003000204020031**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations and conventions.

Q.1 Answer any FIVE from the following questions. **[10]**

(1) Evaluate : $\int_0^\infty x^6 e^{-2x} dx$.

(2) Find the value of $\left|\frac{2}{3}\right|\frac{1}{3}$.

(3) Evaluate: $\int_0^a \int_0^{\sqrt{a^2-x^2}} xydxdy$.

(4) Evaluate: $\int_1^2 \int_0^{\frac{y}{2}} ydydx$.

(5) Find $L\{\text{a sinh } (at)\}$.

(6) Find $L\{t^4 e^{-4t}\}$.

(7) State the second shifting theorem for Laplace transform and inverse Laplace transform.

(8) Evaluate : $L^{-1}\left\{\frac{1}{(p+a)^n}\right\}$ for $n \in N$.

Q.2 **Attempt any TWO.** **[10]**

(1) State and prove the property of Beta function in which the limits of integration are from 0 to ∞.

(2) Show that: $\int_0^2 x(8-x^3)^{\frac{1}{3}}\,dx = \frac{16\pi}{9\sqrt{3}}$.

(3) Show that: $\int_0^\infty e^{-ax}x^{m-1}\cos bx\,dx = \frac{\Gamma m \cos m\theta}{(a^2+b^2)^{\frac{m}{2}}}$ where $\theta = \tan^{-1}\left(\frac{b}{a}\right)$.

Q.3 **Attempt any TWO.** **[10]**

(1) Find the area enclosed between two parabolas $y^2 = 2x$ and $x^2 = 2y$ using double integral.

(2) Evaluate $\iint_S x^2 y\,dx\,dy$ where S is the region bounded by the circle $x^2 + y^2 = a^2$ in the first quadrant.

(3) Change the order of integration of the double integral $\int_0^4 \int_0^{\frac{y}{2}} f(x, y)\,dy\,dx$.

Q.4 **Attempt any TWO.** **[10]**

(1) State and prove the change of scale property for Laplace transform.

(2) State Differentiation of Laplace transform and hence evaluate $\int_0^\infty te^{-3t}\sin t\,dt$.

(3) Evaluate : $\int_0^\infty \frac{e^{-t} - e^{-3t}}{t}\,dt$.

Q.5 **Attempt any TWO.** **[10]**

(1) State and prove the linearity property for Inverse Laplace transform.

(2) Evaluate : $L^{-1}\left[\frac{3p-2}{p^2-4p+20}\right]$

(3) In usual notation prove that $L^{-1}[f(p)] = F(t) \Rightarrow L^{-1}[f(ap)] = \frac{1}{a}F\left(\frac{t}{a}\right)$.